A Coding Strategy for Wireless Networks with no Channel Information

Frédérique Oggier and Babak Hassibi

***Abstract*— In this paper, we present a coding strategy for wireless relay networks, where we assume no channel knowledge. More precisely, the relays operate without knowing the channel that affected their received signal, and the receiver decodes knowing none of the channel paths. The coding scheme is inspired by noncoherent differential space-time coding, and is shown to yield a diversity linear in the number of relays. It is furthermore available for any number of relay nodes.**

I. INTRODUCTION

A lot of attention has been paid recently to coding over wireless networks. Inspired by space-time coding techniques where the transmit antennas cooperate to resist the fading, *cooperative diversity* schemes have been proposed for coding over wireless networks, where the relay nodes form a virtual multiple antennas array to obtain the diversity advantage known to be achieved by MIMO systems [11], [1], [10], [2], [15], [3]. These works have focused on different aspects of wireless networks coding. In [2], the capacity of the network is computed, while in [10], emphasis is put on the pairwise probability of error and the computation of the diversity gain. A lot of work has been done on finding codes optimal with respect to the so-called diversity-multiplexing gain [1], [15], [3]. Some of these works assumed knowledge of the channel both at the relay nodes and at the receiving nodes, some only at the receiving nodes.

In this work, we are interested in designing a strategy where we assume no channel information: the relay nodes do not decode, but just do a simple operation on the received signal for which they do not need to know the fadings, and the receiver decodes with no knowledge of the different paths used during communication. Our strategy is inspired by noncoherent MIMO techniques, in particular the so-called *unitary differential modulation* [5], [7]. Other coding strategies based on differential distributed coding have been proposed independently in [14] and [9].

We organize this paper as follows. In Section II, we start by recalling the wireless network we consider, and how distributed coding is performed. We then present a distributed coding strategy that emulates communication over a noncoherent MIMO channel. This allows us to define a differential coding strategy, described in Section III. In Section IV, we propose a mismatched decoder and show that this yields a diversity gain linear in the number of relays. We conclude with some simulation results.

This work was supported in part by the Swiss National Science Foundation grant PBEL2-110209 and by NSF grant CCR-0133818, by Caltech's Lee Center for Advanced Networking and by a grant from the David and Lucille Packard Foundation.

F. Oggier and B. Hassibi are with the Department of Electrical Engineering, California Institute of Technology, Pasadena, CA 91125. {frederique,hassibi}@systems.caltech.edu

II. NONCOHERENT DISTRIBUTED CODING

A. Distributed Space-Time Codes

Consider a wireless network with $R+2$ nodes which are randomly and independently distributed. Two nodes, a receiver and a transmitter, want to communicate, while the R other nodes serve as relays. Every node is equipped with a single antenna. It can transmit and receive, but not simultaneously. The channel from the transmitter to the ith relay is denoted by f_i, while the one from the ith relay to the receiver is denoted by g_i (see Fig. 1). Both channels are assumed independent complex Gaussian with zero mean and unit variance. The relays do not know the values of the fading coefficients f_i and g_i. We assume a coherence interval of length $T \geq R$ (there is no need to have more relays than coherence time, since it is shown in [10] that the diversity of the system depends on $\min\{T,R\}$). The total power of the system, ρ, is equally distributed between the transmitter and the relays, so that the transmitter has an energy of $P_1=\rho/2$, while each relay has $P_2=\rho/(2R)$. This power allocation was shown [10] to minimize the pairwise probability of error for large number of relay nodes.

The transmission is done in two steps:

Step 1: at the transmitter. Let $\mathbf{s}=(s_1,\ldots,s_T)^t$ be the signal to be sent, from the codebook $\{\mathbf{s}_1,\ldots,\mathbf{s}_L\}$ of cardinality L. The vector $\mathbf{s}$ is normalized such that $\mathrm{E}\mathbf{s}^\dagger\mathbf{s}=1$. Let P_1 be the average power available for every transmission. From time 1 to T, the transmitter sends the signals $\sqrt{P_1T}s_1,\ldots,\sqrt{P_1T}s_T$ to each relay. The received signal at the ith relay at time τ is given by $r_{i,\tau}=\sqrt{P_1T}f_is_\tau+v_{i,\tau}$, with $v_{i,\tau}$ the complex Gaussian noise with zero mean and unit variance:

$$\mathbf{r}_i=\sqrt{P_1T}f_i\mathbf{s}+\mathbf{v}_i,\ i=1,\ldots,R. \tag{1}$$

Step 2: at the relays. From time $T+1$ to $2T$, the ith relay sends $t_{i,1},\ldots,t_{i,\tau}$ to the receiver. The received signal at time $T+\tau$ is given by $y_\tau=\sum_{i=1}^R g_it_{i,\tau}+w_\tau$, where τ is the complex Gaussian noise with zero mean and unit variance:

$$\mathbf{y}=\sum_{i=1}^{R}g_i\mathbf{t}_i+\mathbf{w}. \tag{2}$$

The idea behind *distributed space-time coding* is to design the transmit signal at every relay as a linear function of its received signal:

$$\mathbf{t}_i=\sqrt{\frac{P_2}{P_1+1}}A_i\mathbf{r}_i, \tag{3}$$

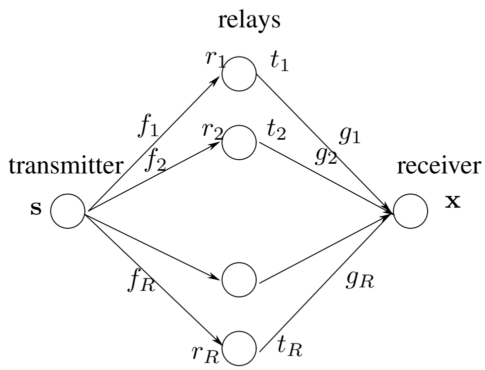

Fig. 1. The wireless network

where A_i is an $T \times T$ matrix, $i = 1, \ldots, R$, and P_2 is the average transmit power for one transmission at every relay node. The normalization factor is chosen so that $E[\mathbf{t}_i^\dagger \mathbf{t}_i] = P_2 T$. In order to have an equitable protocol among different users and among different times instants, the matrices A_i are assumed unitary. This also guarantees that the noise $\mathbf{w}$ at the receiver remains temporally white.

From (2), (3) and (1), the received signal is given by

$$\mathbf{y} = \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} SH + W, \tag{4}$$

with

$$S = [A_1 \mathbf{s} \cdots A_R \mathbf{s}], \; H = \begin{bmatrix} f_1 g_1 \\ \vdots \\ f_R g_R \end{bmatrix} \tag{5}$$

and

$$W = \sqrt{\frac{P_2}{P_1 + 1}} \sum_{i=1}^{R} g_i A_i \mathbf{v}_i + \mathbf{w}.$$

The $T \times R$ matrix S works like a space-time code in a multiple-antenna system. It is called a *distributed space-time code* since it has been generated in a distributed way by the relays.

B. A Noncoherent Channel

So far, the equation

$$\mathbf{y} = \sqrt{\frac{P_1 P_2 T}{P_1 + 1}} SH + W,$$

derived in the previous section, has been considered assuming that H (that is both g_i and f_i) is known [10], [12], [13], [8]. Let us now assume that none of the fadings are known. We a priori consider the same power allocation as in the coherent case. This power allocation minimizes the pairwise error probability based on the Chernoff bound, and we will see in Section IV that we actually get a similar Chernoff bound in our scenario, which validates the same choice of power allocation.

In a traditional noncoherent MIMO setting, it has been argued [4] that the transmitted codeword S has to be unitary. Recall that here

$$S = [A_1 \mathbf{s}, \ldots, A_R \mathbf{s}].$$

The aim is now to design the signal constellation $\{\mathbf{s}_1, \ldots, \mathbf{s}_L\} \ni \mathbf{s}$ and the unitary matrices A_i, $i = 1, \ldots, R$, so that the $T \times R$ matrix S is unitary, i.e, $S^\dagger S = \mathbf{I}_R$. Let

$$\mathbf{s}' = \frac{1}{\sqrt{T}}(1, \ldots, 1)^t, \; \mathbf{s}_i = U_i \mathbf{s}', \; i = 1, \ldots, L,$$

where U_i's are $T \times T$ unitary matrices and $\mathbf{s}'$ is normalized so that $E[\mathbf{s}^\dagger \mathbf{s}] = 1$.

Assume now that there exists a $T \times T$ matrix M such that $MM^\dagger = T$. We can then choose the matrices A_i to be

$$A_i = diag(M_i), \; i = 1, \ldots, R,$$

where M_i denotes a column of M (recall that $T \geq R$). Choosing U_j, $j = 1, \ldots, L$, diagonal makes them commute with all A_i, and we have

$$[A_1 \mathbf{s}_j, \ldots, A_R \mathbf{s}_j] = [U_j A_1 \mathbf{s}', \ldots, U_j A_R \mathbf{s}'] = U_j M / \sqrt{T},$$

and $S^\dagger S = M^\dagger U_j^\dagger U_j M / T = \mathbf{I}_R$ for all transmitted signal $\mathbf{s}_j$. Let us keep in mind that the matrices A_i have to be unitary.

Such matrices M can be found in the class of Butson-Hadamard matrices (for example [6]). Recall that a *Generalized Butson-Hadamard* (GBH) matrix is a $T \times T$ matrix M with coefficients in a ring R such that

$$MM^* = M^* M = T \mathbf{I}_T$$

where M^* is the transpose of the matrix of inverse elements of M: $m_{ij}^* = m_{ji}^{-1}$. If the coefficients of M are chosen to be roots of unity, then $m_{ij}^{-1} = \overline{m_{ij}}$, i.e., the inverse is the conjugate, so that

$$MM^\dagger = M^\dagger M = T \mathbf{I}_T.$$

Furthermore, this implies that all A_i are unitary.

For example, let $\zeta_3 = \exp(2i\pi/3)$ be a primitive 3rd root of unity. Then the matrix

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \zeta_3 & \zeta_3^2 \\ 1 & \zeta_3^2 & \zeta_3 \end{pmatrix}$$

is a Butson-Hadamard matrix. It is easy to check that $MM^\dagger = \mathbf{I}_3$. Also $diag(M_i)$ are unitary, $i = 1, 2, 3$. Other examples of such matrices can be found in [6]. Note that the tensor product of two GBH matrices is again a GBH matrix. This is thus a convenient way of building GBH matrices for a given dimension T.

III. A Differential Coding Strategy

In this section, we give a strategy to implement a differential distributed coding scheme, based on differential MIMO space-time coding, introduced in [5], [7]. It is a priori not clear how to emulate differential coding in a distributed setting. Where should the differential encoding takes places? One can imagine the relays cooperating to encode differentially, similarly to the coherent case where relays encode the space-time codes, as well as having the transmitter itself collaborating with the relays. However, the construction presented in the previous section clearly suggests the approach where the differential encoder is actually at the transmitter itself. The relays cooperate not to encode differentially, but to encode a unitary space-time code.

A. A Differential Encoder

It is straightforward to adapt the two-step transmission described in Subsection II-A to allow for differential encoding and decoding.

Assume that the transmitter wants to send at time $t+nT$ the data z_{t+nT}. It is encoded into a unitary matrix $U(z_{t+nT})$. We consider the following strategy:

1) Let $\mathbf{s}_t = U(z_t)\mathbf{s}'$ be the signal to be transmitted, where $\mathbf{s}' = (1,\dots,1)^t/\sqrt{T}$ is normalized so that $E[\mathbf{s}_t^\dagger \mathbf{s}_t] = 1$. Let P_1 be the average energy available for each transmission. From time $t+1$ to $t+n$, the transmitter sends the signal $\sqrt{P_1 T}\mathbf{s}_t$ to each relay. From time $t+T+1$ to $t+2T$, the signal to be transmitted is $\mathbf{s}_{t+T} = U(z_{t+T})\mathbf{s}_t$.
2) At the ith relay, the received signals are (indexing the signals as a function of the time at which they have been sent)

$$\begin{aligned}\mathbf{r}_i(t) &= \begin{bmatrix} r_{i,1}(t) \\ \vdots \\ r_{i,T}(t) \end{bmatrix} \\ &= \sqrt{P_1 T} f_i \mathbf{s}_t + \mathbf{v}_i(t),\ i=1,\dots,R,\end{aligned}$$

and

$$\begin{aligned}\mathbf{r}_i(t+T) &= \begin{bmatrix} r_{i,1}(t+T) \\ \vdots \\ r_{i,T}(t+T) \end{bmatrix} \\ &= \sqrt{P_1 T} f_i U(z_{t+T})\mathbf{s}_t + \mathbf{v}_i(t+T),\end{aligned}$$

for $i=1,\dots,R$.
3) Each relay $R_i,\ i=1,\dots,R$, multiplies its received signal by a unitary matrix A_i, where A_i has been built using a Butson-Hadamard matrix as described in Subsection II-B. From time $t+T+1$ to $t+2T$, the transmitted signal is $\mathbf{t}_i(t) = \sqrt{\frac{P_2}{P_1+1}}A_i\mathbf{r}_i(t)$, and similarly from time $t+2T+1$ to $t+3T$: $\mathbf{t}_i(t+T) = \sqrt{\frac{P_2}{P_1+1}}A_i\mathbf{r}_i(t+T)$.
4) At time $t+2T$, the received signal is similar to (2) and (4)-(5):

$$\begin{aligned}\mathbf{y}(t) &= \sum_{i=1}^{R} g_i \mathbf{t}_i(t) + \mathbf{w}(t) \\ &= \sqrt{\frac{P_2 P_1 T}{P_1+1}}\sum_{i=1}^{R} g_i f_i A_i \mathbf{s}_t + \qquad (6) \\ &\quad \sqrt{\frac{P_2}{P_1+1}}\sum_{i=1}^{R} g_i A_i \mathbf{v}_i(t) + \mathbf{w}(t).\end{aligned}$$

At time $t+3T$, we have

$$\begin{aligned}\mathbf{y}(t+T) &= \sum_{i=1}^{R} g_i \mathbf{t}_i(t+T) + \mathbf{w}(t+T) \\ &= \sqrt{\frac{P_2 P_1 T}{P_1+1}}\sum_{i=1}^{R} g_i f_i A_i U(z_{t+T})\mathbf{s}_t + \qquad (7) \\ &\quad \sqrt{\frac{P_2}{P_1+1}}\sum_{i=1}^{R} g_i A_i \mathbf{v}_i(t+T) + \mathbf{w}(t+T).\end{aligned}$$

Under the assumption that A_i and $U(z_{t+T})$ commute, for all i and for all possible choices of $U(z_{t+T})$, we can plug equation (7) into equation (8), which yields

$$\mathbf{y}(t+T) = U(z_{t+T})\mathbf{y}(t) + W(t,t+T), \qquad (8)$$

where

$$W(t,t+T) = -U(z_{t+T})\sqrt{\frac{P_2}{P_1+1}}\sum_{i=1}^{R} g_i \mathbf{v}_i(t)$$

$$-U(z_{t+T})\mathbf{w}(t) + \sqrt{\frac{P_2}{P_1+1}}\sum_{i=1}^{R} g_i \mathbf{v}_i(t+T) + \mathbf{w}(t+T)$$

is the noise term. Note that the channel coefficients f_i and g_i do not appear in (8). Also, the assumption that A_i and $U(z_{t+T})$ is valid since both the unitary codewords U_j and the matrices A_i are chosen diagonal.

IV. Decoding and Diversity

Emulating the point to point case, a natural candidate for the differential decoder is

$$\arg \min_{U_l,\ l=1,\dots,L} \|\mathbf{y}(t+T) - U_l\mathbf{y}(t)\|^2.$$

Let us restrict to the case where $T=R$. In order to analyze this strategy, we consider two instances of the noncoherent channel,

$$\mathbf{y}(t) = \sqrt{c_\rho}\mathbf{X}(t)\mathbf{H} + \mathbf{w}(t)$$

where $\mathbf{H}$ is an $T\times T$ matrix unknown at both the transmitter and receiver, $\mathbf{X}(t)$ is a $T\times T$ unitary matrix, and c_ρ is a constant which depends on the SNR ρ, that is,

$$\begin{pmatrix} \mathbf{y}(t) \\ \mathbf{y}(t+T) \end{pmatrix} = \sqrt{c_\rho}\begin{pmatrix} \mathbf{X}(t) \\ \mathbf{X}(t+T) \end{pmatrix}\mathbf{H} + \begin{pmatrix} \mathbf{v}(t) \\ \mathbf{v}(t+T) \end{pmatrix}.$$

Since $\mathbf{X}(t)$ and $\mathbf{X}(t)\psi$ are indistinguishable for an arbitrary unitary $T\times T$ matrix ψ, we preprocess the signal so that

$$\begin{pmatrix} \mathbf{y}(t) \\ \mathbf{y}(t+T) \end{pmatrix} = \sqrt{c_\rho}\begin{pmatrix} \mathbf{I}_T \\ U_k \end{pmatrix}\mathbf{H} + \begin{pmatrix} \mathbf{v}(t) \\ \mathbf{v}(t+T) \end{pmatrix},$$

for U_k a unitary matrix belonging to the codebook. To suit the network model, we have

$$\mathbf{v}(t) = \sqrt{\frac{P_2}{P_1+1}}\sum_{i=1}^{T} A_i g_i \mathbf{v}_i + \mathbf{w}(t)$$

and $\mathbf{H} = D_g\mathbf{f}$, where $D_g = diag(g_1,\dots,g_T)$ and $\mathbf{f} = (f_1,\dots,f_T)^t$. Furthermore, we have $c_\rho = \frac{P_2 P_1 T}{P_1+1}$, and we denote $c'_\rho = \frac{P_2}{P_1+1}$. Recall that $P_1 = \rho/2$, $P_2 = \rho/(2R)$ and

ρ is the total power of the system. Because of the two steps transmission, both the noise $\mathbf{v}(t)$ and the channel matrix $\mathbf{H}$ contains products of Gaussian random variables, which makes a precise analysis difficult. In this work, we thus consider a mismatched decoder, and we will show that such a decoder already gives the diversity.

A. The Pairwise Error Probability of the Mismatched Decoder

Let us now compute the pairwise error probability of decoding with a mismatched decoder. Knowing $\mathbf{g} = (g_1, \ldots, g_T)^t$, we have

$$\begin{aligned}
&E[\mathbf{v}(t)\mathbf{v}(t)^\dagger] = E[\mathbf{v}(t+T)\mathbf{v}(t+T)^\dagger] \\
&= c'_\rho E[\textstyle\sum_{i,j=1}^T A_i g_i \mathbf{v}_i (A_j g_j \mathbf{v}_j)^\dagger] + E[\mathbf{w}(t)\mathbf{w}(t)^\dagger] \\
&= c'_\rho \textstyle\sum_{i=1}^T A_i A_i^\dagger |g_i|^2 \mathbf{I}_T + \mathbf{I}_T \\
&= (c'_\rho \|\mathbf{g}\|^2 + 1)\mathbf{I}_T =: a\mathbf{I}_T.
\end{aligned}$$

Let $\mathbf{y} = [\mathbf{y}(t) \;\; \mathbf{y}(t+T)]^t$, we have

$$\Sigma := E[\mathbf{y}\mathbf{y}^\dagger] = \left(\begin{array}{cc} c_\rho D_{|g|} + a\mathbf{I}_T & c_\rho D_{|g|} U_k^\dagger \\ c_\rho D_{|g|} U_k & c_\rho D_{|g|} + a\mathbf{I}_T \end{array} \right),$$

where $D_{|g|} = diag(|g_1|^2, \ldots, |g_T|^2)$.

The pairwise probability of error $P(U_k \to U_l)$ is given by

$$\begin{aligned}
&P(\|\mathbf{y}(t+1) - U_k\mathbf{y}(t)\|^2 \geq \|\mathbf{y}(t+1) - U_l\mathbf{y}\|^2 \mid U_k \text{ is sent}) \\
&= P(\|[-U_k \;\; \mathbf{I}_T]\; \mathbf{y}\|^2 \geq \|[-U_l \;\; \mathbf{I}_T]\; \mathbf{y}\|^2 \mid U_k \text{ is sent}) \\
&= P(\mathbf{y}^\dagger \mathbf{U} \mathbf{y} \geq 0 \mid U_k \text{ is sent})
\end{aligned}$$

with $k \neq l$ and

$$\mathbf{U} = \left(\begin{array}{cc} 0 & U_l^\dagger - U_k^\dagger \\ U_l - U_k & 0 \end{array} \right). \tag{9}$$

We now compute $P(\mathbf{y}^\dagger\mathbf{U}\mathbf{y} \geq 0)$ knowing $\mathbf{g}$. We have

$$\begin{aligned}
P(\mathbf{y}^\dagger\mathbf{U}\mathbf{y} \geq 0 |\; \mathbf{g}) &= E[u(\mathbf{y}^\dagger\mathbf{U}\mathbf{y})] \\
&= E\left[\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{i\omega\mathbf{y}^\dagger\mathbf{U}\mathbf{y}}}{i\omega} d\omega \right]
\end{aligned}$$

where u is the step function ($u(x) = 1$ if $x > 0$, $u(x) = 0$ else), and the second equality is the Fourier transform of u. Computing the expectation yields

$$\begin{aligned}
&\tfrac{1}{2\pi} \int\int_{-\infty}^{\infty} \tfrac{e^{i\omega\mathbf{y}^\dagger\mathbf{U}\mathbf{y}}}{i\omega} \tfrac{e^{-\mathbf{y}^\dagger\Sigma^{-1}\mathbf{y}}}{(2\pi)^{2T}\det(\Sigma)} d\omega dy \\
&= \tfrac{1}{(2\pi)^{2T+1} i} \int_{-\infty}^{\infty} \int \tfrac{e^{-\mathbf{y}^\dagger(-i\omega\mathbf{U} + \Sigma^{-1})\mathbf{y}}}{\omega\det(\Sigma)} dy d\omega.
\end{aligned}$$

Since the exponent of the exponential is of the form $i\mathbf{y}^\dagger\omega\mathbf{U}\mathbf{y} - \mathbf{y}^\dagger\Sigma^{-1}\mathbf{y}$, with real part $-\mathbf{y}^\dagger\Sigma^{-1}\mathbf{y}$ which is negative, and imaginary part given by $i\mathbf{y}^\dagger\omega\mathbf{U}\mathbf{y}$ (recall that $\mathbf{U}$ is Hermitian), then this integral converges and we have

$$P(\mathbf{y}^\dagger\mathbf{U}\mathbf{y} \geq 0 |\; \mathbf{g}) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{1}{\omega\det(\mathbf{I} - i\omega\mathbf{U}\Sigma)} d\omega$$

We have that the above determinant is given by

$$\begin{aligned}
&\textstyle\prod_{k=1}^T (1 + i\omega c_\rho |g_k|^2 |u_{ik} - u_{jk}|^2 + \\
&\omega^2 |u_{ik} - u_{jk}|^2 (2ac_\rho |g_k|^2 + a^2)),
\end{aligned}$$

where $a = c'_\rho \|\mathbf{g}\|^2 + 1$. Since our goal is a diversity computation, we are interested in an very high SNR regime. Note that when ρ is big, $c'_\rho = \frac{P_2}{P_1+1} = \frac{\rho}{T(\rho+2)} \to 1/T$. Since g was drawn $\mathcal{CN}(0,1)$, $c'_\rho\|g\|^2 + 1$ is bounded between 1 and 2, so that we have

$$\prod_{k=1}^T c_\rho |g_k|^2 |u_{ik} - u_{jk}|^2 \left[\frac{1}{c_\rho |g_k|^2 |u_{ik} - u_{jk}|^2} + i\omega + \omega^2 2a \right].$$

By completing the squares, we get that $2\pi i P(\mathbf{y}^\dagger\mathbf{U}\mathbf{y} \geq 0 |\; \mathbf{g})$ is given by

$$\int_{-\infty}^{\infty} \frac{1}{\omega} \left[\prod_{k=1}^T c_\rho |g_k|^2 |u_{ik} - u_{jk}|^2 2a \left[\left(\omega + \frac{i}{4a} \right)^2 + c_k^2 \right] \right]^{-1} d\omega$$

where

$$c_k := \sqrt{\frac{1}{16a^2} + \frac{1}{c_\rho |g_k|^2 |u_{ik} - u_{jk}|^2}}.$$

Note that the above integral has poles in $\omega = -i(1/4a \pm c_k)$. Thus as long as $= -i(1/4a - c_k) < Im(\omega) < -i(1/4a + c_k)$, the above integral is well-defined. We thus choose the following contour of integration, within the convergence region

$$\int_{-\infty - \frac{i}{4a}}^{\infty - \frac{i}{4a}} \frac{1}{\omega} \left[\prod_{k=1}^T c_\rho |g_k|^2 |u_{ik} - u_{jk}|^2 2a \left[\left(\omega + \frac{i}{4a} \right)^2 + c_k^2 \right] \right]^{-1} d\omega$$

and with a change of variable, we get

$$\int_{-\infty}^{\infty} \frac{1}{\omega - \frac{i}{4a}} \left(\prod_{k=1}^T c_\rho |g_k|^2 |u_{ik} - u_{jk}|^2 2a \left[\omega^2 + c_k^2 \right] \right)^{-1} d\omega$$

Following [4], we obtain a bound on the probability of error that we know real by taking the real part of the above expression:

$$\begin{aligned}
&P(\mathbf{y}^\dagger\mathbf{U}\mathbf{y} \geq 0 |\; \mathbf{g}) \\
&\lessapprox \tfrac{1}{2\pi} \int_{-\infty}^{\infty} \tfrac{4a}{1+16a^2\omega^2} (\textstyle\prod_{k=1}^T c_\rho |g_k|^2 |u_{ik} - u_{jk}|^2 2a[\omega^2 \\
&\qquad\qquad + \tfrac{1}{16a^2}] + 1)^{-1} d\omega \\
&\leq \tfrac{1}{2\pi} \int_{-\infty}^{\infty} \tfrac{4ad\omega}{1+16a^2\omega^2} \left(\textstyle\prod_{k=1}^T c_\rho |g_k|^2 |u_{ik} - u_{jk}|^2 \tfrac{1}{8a} + 1 \right)^{-1} \\
&= \tfrac{1}{2} \left(\textstyle\prod_{k=1}^T c_\rho |g_k|^2 |u_{ik} - u_{jk}|^2 \tfrac{1}{8a} + 1 \right)^{-1} \\
&= \tfrac{1}{2} \det(\mathbf{I}_T + \tfrac{c_\rho}{8(c'_\rho\|g\|^2+1)} D_{|g|} (U_k - U_l)(U_k - U_l)^\dagger)^{-1}.
\end{aligned}$$

Thus

$$\begin{aligned}
&P(U_k \to U_l) = P(\mathbf{y}^\dagger\mathbf{U}\mathbf{y} \geq 0) \\
&\leq E_{\mathbf{g}} \det(\mathbf{I}_T + \tfrac{c_\rho}{8(c'_\rho\|g\|^2+1)} D_{|g|} (U_k - U_l)(U_k - U_l)^\dagger)^{-1}.
\end{aligned}$$

This bound on the pairwise probability of error is similar to the Chernoff bound obtained in [10, Theorem 1], where it has been proven, by computing the above expectation on g_i, that the diversity gain is given by rank$((U_k - U_l)(U_k - U_l)^\dagger) \left(1 - \frac{\log\log P}{logP} \right)$. Thus when $(U_k - U_l)(U_k - U_l)^\dagger$ is full rank (that is, the code is *fully diverse*), we get a diversity of

$$R\left(1 - \frac{\log\log P}{\log P} \right). \tag{10}$$

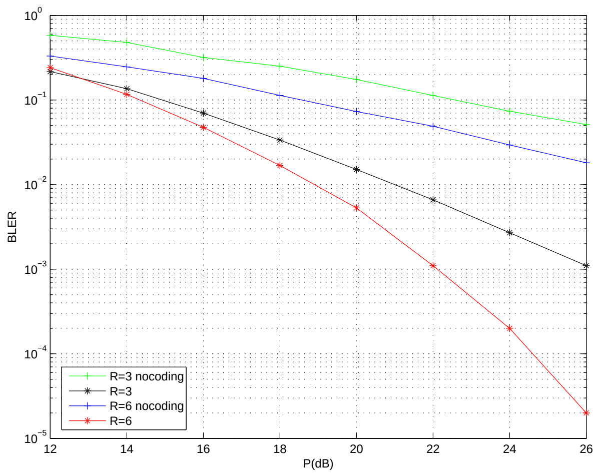

Fig. 2. BLER as a function of ρ, the power of the system in dBs, for $R = 3, 6$ relays, with and without coding at the relays.

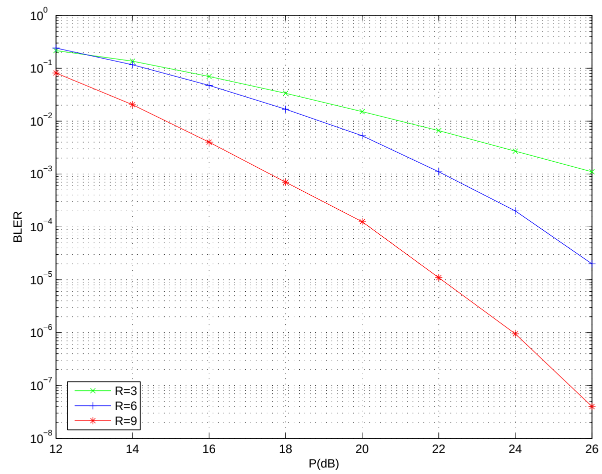

Fig. 3. BLER as a function of ρ, the power of the system in dBs, for $R = 3, 6, 9$ relays.

V. Code Constructions

To conclude, we provide some code constructions with simulation results. The code design consists in constructing the unitary matrices $U(z)$ in which the data z to be sent will be encoded. According to the pairwise error probability bound, these matrices have to be fully diverse. Since it is required for the differential encoding to have matrices $U(z)$ that commute with A_i, we thus choose the matrices $U(z)$ to be diagonal unitary matrices. Such matrices have already been studied for small dimensions in [5].

Let us illustrate the code construction with 3 relay nodes. Let $\zeta_{63} = \exp(2i\pi/63)$. The codebook is given by

$$\left\{ D^i,\ D = \begin{pmatrix} \zeta_{63} & 0 & 0 \\ 0 & \zeta_{63}^{17} & 0 \\ 0 & 0 & \zeta_{63}^{26} \end{pmatrix} \right\}$$

for $i = 1, \dots, 63$. The relays A_i, $i = 1, 2, 3$, use respectively the matrices

$$A_1 = \mathbf{I}_3,\ A_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \zeta_3 & 0 \\ 0 & 0 & \zeta_3^2 \end{pmatrix},\ A_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \zeta_3^2 & 0 \\ 0 & 0 & \zeta_3 \end{pmatrix}.$$

Simulation results are reported in Fig. 2 and 3. They show the BLER as a function of ρ, the power of the system in dB.

In Fig. 2, simulation results are shown for $R = 3, 6$, relay nodes. We compare the performance of the same codebook with and without coding at the relays. As expected, it is obvious that no coding at the relays yield no diversity, and the curves for $R = 3$ and $R = 6$ are parallel.

In Fig. 3, we are interested in the gain of diversity obtained by adding relays. The diversity should increase linearly with the number of nodes. A clear diversity gain is obtained going from 3, to 6, then 9 relays.

VI. Conclusion

We considered the problem of coding over a wireless network. While existing schemes heavily rely on the knowledge of the channel, either at both relays and receiver, or at least at the receiver, we presented a scheme that requires no channel knowledge. This scheme is available for any number of relay nodes, and its analysis shows that it yields a diversity which is linear in the number of relays.

VII. Acknowledgments

The authors would like to thank Y. Jing for sending her preprint.

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